Introduction to Plasma Physics

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This part of the course is based on Refs. [1], [2] and [3].

1. What is plasma ?

One of the most fundamental forces around us is the **electromagnetic force**. Often in nature, this force creates **structures**: molecules, crystalline solids, etc. Structures systems have binding energies larger than the ambient thermal energy. Indeed, placed in sufficiently hot environment, the structures decompose - solids melt, molecules disassociate etc.

At even higher temperatures, atoms decompose into electrons and ions. However, these charged particles are not free: they are affected by each other's electromagnetic fields. Nonetheless, as the charges are no longer bound, they show various forms of complex *collective* motion. This collection of charged particles is termed **plasma**.

Since the inter-atomic bonds are weaker than the atomic bonds, as the ambient temperature increases, they break before the atoms gets ionized. Thus, most terrestrial plasmas begin as gases. In fact, sometimes a plasma is defined as a gas that is sufficiently ionized to exhibit plasma-like behavior. Note though that often partial ionization is sufficient to produce most characteristics of fully ionized gases.

When plasmas are formed by ionizing neutral gas, they are **quasi-neutral**: they contain equal number of electrons and ions. The "quasi" originates from the small deviation from exact neutrality due to the motion of the charges, and often have far-reaching consequences. Strongly non-neutral plasmas occur mainly in laboratory experiments. Their equilibrium depends on the existence of strong magnetic fields.

Plasma as a state of matter is extremely abundant in the universe: more than 95% of the known matter in the universe (stars, interstellar medium, solar winds, etc.) is in the form of plasma. Plasma is also abundant on earth: it can be found in lightning, fluorescent lamps, and many industrial processes, including many liquids and even solid-state systems who sometimes show some characteristics of plasma.

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2. History and relevance

The term "Plasma" was used since the 18^{th} century in medicine - to describe the transparent liquid that remains after a blood is cleared from its various corpuscles. This term was first used in 1927 by **Irving Langmuir** to describe ionized gas. Indeed Langmuir (who won the 1932 Nobel prize in Chemistry) pioneered in many ways the study of plasma physics.

Following the pioneering work of Langmuir, plasma physics spread into several directions.

1. The development of radio broadcast led to the discovery of the earth's **ionosphere**, a layer of partially ionized gas in the upper atmosphere which reflects radio waves. This layer is responsible for the fact that transmitted radio signals can be received beyond the horizon.

2. In astrophysics, most objects of interest - in fact, nearly all of the universe, consists of plasma. The study of astrophysical plasmas was pioneered by **Hannes Alfvén** (1970 Nobel prize in Physics), who developed the theory of **magneto-hydrodynamics** (MHD), in which the plasma is treated as a conducting fluid. This theory had been successfully implemented in the study of many astronomical objects, such as solar flares, sun spots, star formation and many others. Two topics of particular interest are **magnetic reconnection** - topological change of magnetic field lines that convert magnetic energy into thermal energy (and acceleration of particles), and **dynamo** which explains how motion of MHD fluid can generate strong, macroscopic magnetic fields (without it, both the sun and the earth would lose their magnetic field very quickly).

3. Following the detonation of the hydrogen bomb in 1952, a great deal of interest began in **controlled thermonuclear fusion** as a possible (unlimited) power source. The basic reaction involved deuterium (D) and tritium (T) atoms, are as follows:

$$
\begin{aligned}
D + D &\to {}^3He + n + 3.2 \text{ MeV} \\
D + D &\to T + p + 4.0 \text{ MeV} \\
D + T &\to {}^4He + n + 17.6 \text{ MeV}.
\end{aligned}
$$

   The cross section for these fusion reactions are appreciable only for incident energies above 5 keV. The main problem is to heat and maintain a plasma at ~ 10 keV, and is still unsolved.

   After the subject became declassified in 1958, many important papers were published in the late 1950's - early 1960's, which mark the beginning of modern plasma physics. Fusion physicists are mostly concerned with understanding how a thermonuclear plasma can be trapped (mostly by a magnetic field), and the instabilities which may allow it to escape.

4. In 1958, **James Van Allen** discovered the radiation belts surrounding earth (that are named after him - the Van Allen belts). These belts are made of energetic charged particles (mostly originated from the solar wind) that are captured by and held around a planet by the planet's magnetic field. This discovery marks the first systematic study of the earth's magnetosphere via satellites, and opened the field of **space plasma physics**.

5. The development of high power lasers in the 1960's opened up the field of **laser plasma physics**. When a high powered laser beam strikes a solid target, material is immediately ablated, and a plasma is formed at the boundary between the beam and the target. Laser plasmas often have fairly extreme properties (e.g., very high densities), which enable to study the properties of plasma under extreme conditions.

3. The defining quantities of a plasma state

As stated above, a plasma is a mixture of positive ions and electrons. A plasma can be fully ionized (e.g., the plasma in the sun) or partially ionized (e.g., in fluorescent lamps). However, in order to fully define the state of a plasma, we need to specify a few additional ingredients, such as density, temperature, Debye length and plasma parameter.

3.1. State of matter

Let us begin by considering a *neutral* gas. The gas is characterized by the number of particles per unit volume, which is called the **number density** and is denoted by n. In MKS units which we will use in this course, the number density has units of m^{-3}.

In thermodynamic equilibrium, the particles motion is described by the **temperature** (T) of the gas. In ideal gas, the product of the temperature and the number density gives the pressure, via $p = nk_BT$, where $k_B = 1.38 \times 10^{-23}$ m^2 kg s^{-2} °K^{-1} (or $J/°K$) is Boltzmann's constant.

When considering plasma, there is a mixture of two different gases: heavy ions and light electrons. We thus distinguish between the properties of the electron and ion gases, by using different terms for the densities, n_i and n_e and the temperatures, T_i and T_e. In many cases the plasma is isothermal and $T_i = T_e$ (e.g., in the interior of the sun), but this is not always the case.

3.2. The Boltzmann distribution and the concept of temperature

Before proceeding further, let us remind ourselves a few elementary concepts from classical statistical mechanics. When a gas is in thermodynamic equilibrium having temperature T, the relative population of two energy states E_j and E_l [2] is given by

$$\frac{p_j}{p_l} = \frac{g_j}{g_l} \exp\left(-\frac{E_j - E_l}{k_B T}\right), \tag{1}$$

where g_j and g_l are the degeneracies of states j and l (the number of sub-states with the same energy).

Derivation of Boltzmann distribution appears in many statistical physics textbooks, and so there is no point repeating it here.

As a specific example, we have the Maxwell-Boltzmann (or Maxwellian) velocity distribution of free particles of mass m,

$$f(v_x, v_y, v_z) = A \exp\left[-\frac{\frac{m}{2}(v_x^2 + v_y^2 + v_z^2)}{k_B T}\right], \tag{2}$$

where A is a normalization factor. Here, $f(v_x, v_y, v_z)dv_x dv_y dv_x$ is the number density of particles having velocity in the range $v_x..v_x + dv_x$, $v_y..v_y + dv_y$, $v_z..v_z + dv_z$. The total number density n is thus given by

$$n = \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z f(v_x, v_y, v_z). \tag{3}$$

Using the number density, one can find the normalization factor,

$$A = n \left(\frac{m}{2\pi k_B T}\right)^{3/2}. \tag{4}$$

The average kinetic energy per particle is calculated via

$$\langle E_k \rangle = \frac{\int_{-\infty}^{\infty} \frac{1}{2} m \left(v_x^2 + v_y^2 + v_z^2\right) f(v_x, v_y, v_z) dv_x dv_y dv_z}{\int_{-\infty}^{\infty} f(v_x, v_y, v_z) dv_x dv_y dv_z} = \frac{3}{2} k_B T, \tag{5}$$

which is consistent with the general result that $\langle E_k \rangle$ is equal to $\frac{1}{2} k_B T$ per degree of freedom.

Because of the close relation between temperature and average kinetic energy, it is common in plasma physics to give temperature units of energy; in particular, 1 eV $= 11,600$°K.

[2] not to be confused with the electric field.

4. Debye shielding

A fundamental difference between plasma and neutral gas, is that in neutral gas particles interact only during collisions. In practice, the atoms or molecules in the gas interact with each other via short-range van-der-Waals forces, which decay with distance as r^{-6}. As opposed to that, charged particles in plasma interact via long-range Coulomb force, which decays as r^{-2}. This means that each plasma particle interacts simultaneously with a large number of other particles, leading to a macroscopic result- a **collective behavior**.

The first important example of this behavior is the ability of plasma to shield out electric potentials that are applied to it. Supposed we put an electric field inside the plasma by inserting two charged balls connected to a battery (see Figure 1). The balls would attract particles of the opposite charge, and a cloud of ions would immediately surround the negative ball, while a cloud of electrons would surround the positive ball.

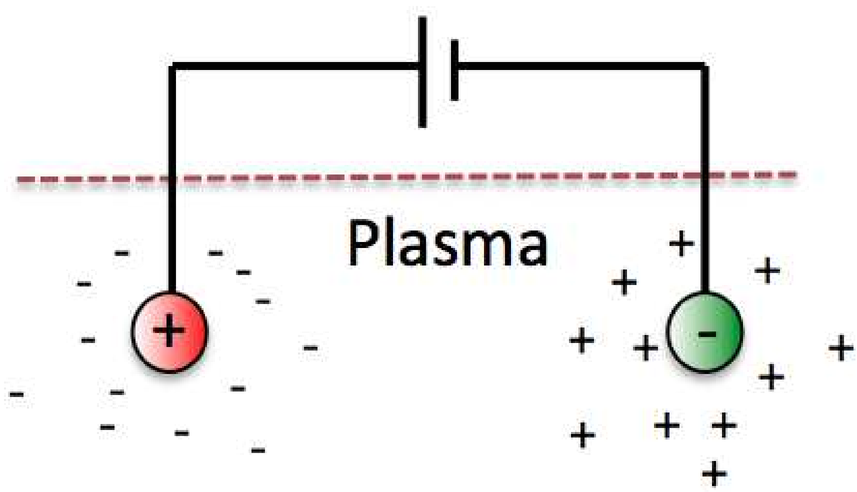

Fig. 1.— Debye shielding.

If the plasma were cold, and there were no thermal motion, the number of charges in the cloud would balance exactly the number of charges in the ball - the shielding would be perfect, and no electric field would be present in the plasma outside the clouds.

However, for a finite temperature, particles at the edge of the cloud (where the electric field is weak) have enough thermal energy to escape the electrostatic potential well. Thus, the "edge" of the cloud is at radius where the potential energy is approximately equal to the thermal energy, $k_B T$ of the particles.

Let us calculate the approximate thickness of this cloud of charged particles. Consider an electric potential $\Phi = \Phi_0$ at $\vec{r} = 0$. We want to compute $\Phi(\vec{r})$ (see Figure 2). We write Poisson's equation,

$$\epsilon \vec{\nabla}^2 \Phi = -\rho. \tag{6}$$

Here, ϵ is the permittivity of the medium (in free space, $\epsilon_0 = 8.85 \times 10^{-12}$ F/m in MKS units), and ρ is the **charge density**.

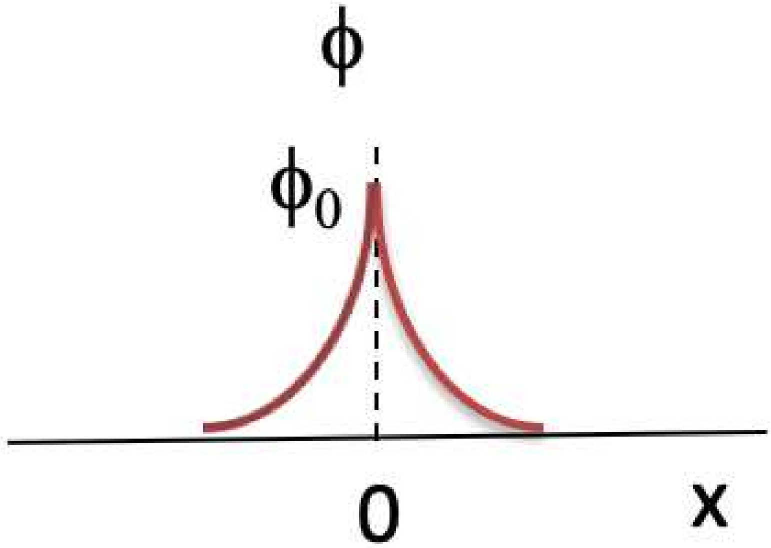

Fig. 2.— Potential distribution in a plasma

In the problem at hand, the space charge consists of a point charge Q at $r = 0$ and the contribution from the electrons and ions, which enables to write

$$\triangle \Phi = -\frac{\rho}{\epsilon} = -\frac{1}{\epsilon}\left[Q\delta(\vec{r}) - qn_e(\vec{r}) + qn_i(\vec{r})\right], \tag{7}$$

where q is the electron's charge, and the charge density due to the electrons and ions in the plasma is $\rho = q(n_i - n_e)$.

Far away from the source (at $|\vec{r}| \to \infty$), $n_i = n_e = n_\infty$. In the presence of an attractive electric potential with potential energy $q\Phi$ (the potential is caused by a positive charge, $+Q$), the number of particles in a thermal distribution that are found in the vicinity of the potential is given using the Boltzmann factor (Equation 1),

$$\begin{aligned} n_e(\vec{r}) &= n_{e,\infty} \exp\left(+\frac{q\Phi(\vec{r})}{k_B T_e}\right), \\ n_i(\vec{r}) &= n_{i,\infty} \exp\left(-\frac{q\Phi(\vec{r})}{k_B T_i}\right). \end{aligned} \tag{8}$$

For simplicity, we assume that the perturbed electric potential energy $|q|\Phi$ is small compared to thermal energy. We can then use Taylor expansion,

$$\begin{aligned} n_e(\vec{r}) &\approx n_{e,\infty}\left(1+\tfrac{q\Phi(\vec{r})}{k_B T_e}\right), \\ n_i(\vec{r}) &\approx n_{i,\infty}\left(1-\tfrac{q\Phi(\vec{r})}{k_B T_i}\right). \end{aligned} \tag{9}$$

Using the linearized term in Poisson's equation (7), and the fact that $n_{e,\infty} = n_{i,\infty}$, we get

$$\triangle\Phi = -\frac{1}{\epsilon}\left[Q\delta(\vec{r}) - qn_{e,\infty}\frac{q\Phi(\vec{r})}{k_B T_e} - qn_{i,\infty}\frac{q\Phi(\vec{r})}{k_B T_i}\right] \tag{10}$$

Rearranging all contributions that contain Φ in the left hand side, and using the spherical symmetry of the problem, we obtain a Helmholtz-type equation,

$$\frac{\partial^2\Phi}{\partial r^2} + \frac{2}{r}\frac{\partial\Phi}{\partial r} - \frac{1}{\lambda_D^2}\Phi = -\frac{Q}{\epsilon_0}\delta(\vec{r}). \tag{11}$$

The parameter λ_D has the dimension of length and is defined by

$$\frac{1}{\lambda_D^2} = \frac{q^2 n_{e,\infty}}{\epsilon k_B T_e} + \frac{q^2 n_{i,\infty}}{\epsilon k_B T_i}. \tag{12}$$

The solution to the Helmholtz equation is

$$\Phi(\vec{r}) = \frac{Q}{4\pi\epsilon r}e^{-r/\lambda_D}. \tag{13}$$

(This is known as Debye-Hückel potential).

The parameter λ_D is the **Debye shielding length** (or **Debye length** for short) which describes the combined shielding of electrons and ions. Defining $n_\infty = n_{e,\infty} + n_{i,\infty}$, for $T_e = T_i$ we can write

$$\lambda_D = \left(\frac{\epsilon k_B T_e}{n_\infty q^2}\right)^{1/2}. \tag{14}$$

As the temperature rises, the shielding length increases - due to the thermal motion of the particles.

4.1. Quasi-neutrality

The Debye length represents a natural length scale that describes deviations from neutrality in the plasma. We can now define "quasineutrality" of the plasma. If the dimension L

of the system is much larger than λ_D, then whatever local concentration of charge may arise (or external potentials are introduced into the system), these are shielded out in a distance short compared with L. Thus, the bulk of the plasma is free of large electric potentials or fields, and we say that the plasma is **quasi-neutral**.

Alternatively, we can write that the criterion for an ionized gas to be a plasma is that it is dense enough such that $\lambda_D \ll L$.

5. The plasma parameter, plasma frequency and criteria for plasmas

Clearly, the Debye shielding effect can be considered as a valid effect only as long as there is sufficient number of particles in the plasma. One can compute the number of particles in a Debye sphere,

$$N_D = \frac{4}{3}\pi\lambda_D^3 n = 1.38 \times 10^6\ T^{3/2}/n^{1/2} \quad (T \text{ in } ^\circ K). \tag{15}$$

One can therefore add the requirement that $N_D \gg 1$ for the calculation to be meaningful.

Another quantity of interest is the response time to an external electric field. When the potential perturbation is small, $|q\Phi| \ll k_B T$, the electron energy is not much changed from its thermal value. Hence, the typical electron velocity remains close to its thermal velocity, $v_{th,e} \approx (k_B T_e/m_e)^{1/2}$.

For an equilibrium to be reached, the electrons need to travel a typical distance λ_D. The time it takes them to do so can be estimated by $\tau \approx \lambda_D/v_{th,e}$. The reciprocal of this response time is called the **electron plasma frequency**,

$$\omega_{p,e} = \frac{v_{th,e}}{\lambda_D} = \left(\frac{n_\infty q^2}{\epsilon m_e}\right)^{1/2} \tag{16}$$

(We will further discuss this quantity in details below).

This line of reasoning provides us with 3 requirements for a gas to behave like a plasma, rather than a neutral gas:

$$\begin{array}{ll} 1. & \lambda_D \ll L, \\ 2. & N_D \gg 1, \\ 3. & \omega_{p,e} T \gg 1. \end{array} \tag{17}$$

Here, L is the typical size of the system, and T is the typical time scale.

6. Existence regimes

Plasmas are found in nature in a huge parameter space, covering about 7 orders of magnitude in temperature, and more than 25 orders of magnitude in (electron) density (see Figure 3). In the figure, marked are several examples of plasmas of interest.

- The green region in the bottom right marks the transition to the regime where the density is high enough such that $N_D < 1$. This regime is called "strong coupling".
- When the average distance between the particles become smaller than the de-Broglie wavelength, $\lambda_{dB} = h/(m_e v_{th,e})$, quantum effects become important, and we must use the Fermi-Dirac statistics. In this case, the plasma is called **degenerate**. The conditions where such a plasma is found are within the strongly coupled regime, and are typical for dead stars, such as white dwarfs. Another example is electrons in a metal, which form a strongly coupled degenerate system.
- Relativistic effects become important at temperatures $T > 10^9$ °K, as the thermal velocity becomes comparable to the speed of light, c. At higher temperatures, the plasma becomes *relativistic*.
- At temperatures $T \leq 10^5$ °K, recombination of electrons and ions can be significant, and the plasmas are often only partially ionized.

REFERENCES

[1] F. Chen, *Introduction to Plasma Physics and Controlled Fusion* (Springer), chapter 1.

[2] A. Piel, *Plasma Physics: An Introduction to Laboratory, Space, and Fusion Plasmas* (Springer), chapters 1 and 2.

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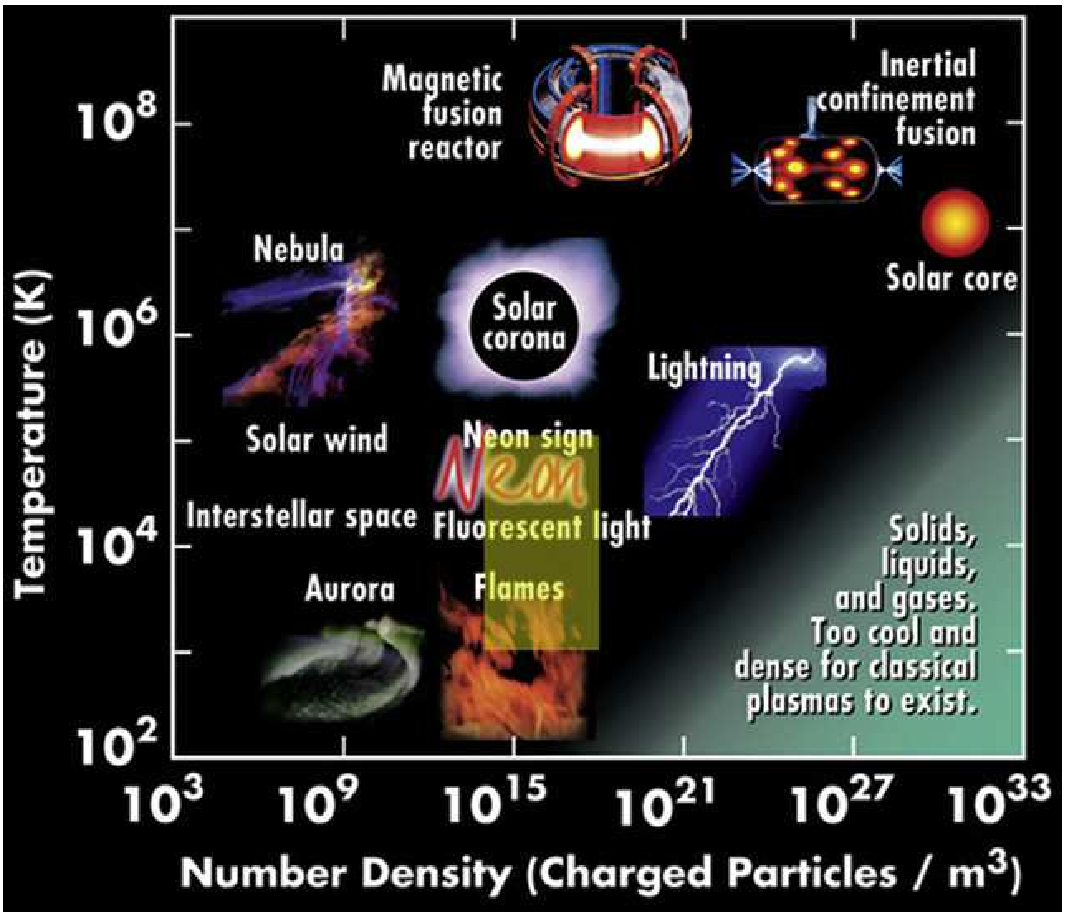

Fig. 3.— Existence diagram of various plasmas. Reference: Cardinaud, C., 2018, "Fluorine-based plasmas: Main features and application in micro-and nanotechnology and in surface treatment", C. R. Chimie 21 (2018) 723 - 739