Introduction: what is quantum field theory ?

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This part of the course is based on Ref. [1]

1. Relativistic quantum mechanics

By the mid 1920's, the basics of **quantum mechanics** were already discovered by people like *de Broglie, Bohr, Schrödinger, Pauli* and *Heisenberg*. However, in its basic form, quantum mechanics is **inconsistent** with the already known theory of **special relativity**. There are various ways to see this: for example, the amplitude for a free particle to propagate from $\vec{x}_0$ to $\vec{x}$ is

$$U(t) = \langle \vec{x} | e^{-iHt} | \vec{x}_0 \rangle, \tag{1}$$

which is nonzero for **all** $\vec{x}$ and t, indicating that a particle can propagate between any two points at arbitrary short time; this is in contradiction, of course, to the requirement set by special relativity, that nothing travels faster than the speed of light, c. Thus, Schrödinger equation needs to be replaced by a relativistic equation (known as *Klein-Gordon* equation).

A different, yet related question is the question of **particle interaction**. Schrödinger equation, for example, explains how a single particle evolves under the influence of an external potential; however, it does not describe interactions between particles, such as electron-photon scattering (*Compton* scattering), or the production of new particles that take place when relativistic particles interact. A classical example is the prediction of the anti-electron particle (the **positron**) by *Dirac* in 1928, that results when two energetic photons interact ($\gamma + \gamma \rightarrow e^+ + e^-$). The positron was discovered by *Anderson* in 1932. In following years, a plethora of new particles were discovered.

2. The electron-photon duality

The concept of wave-particle duality tells us that the properties of electrons and photons are fundamentally very similar. Despite obvious differences in their mass and charge, under the right circumstances both suffer wave-like diffraction and both behave like particle.

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Yet the appearance of these objects in classical physics is very different. **Electrons** and other matter particles are postulated to be **elementary constituents of Nature**. In contrast, **light** is a **derived concept**: in classical EM, the fundamental entities are the electromagnetic fields. Maxwell thought of photons as a ripple of these fields.

If photons and particles are truely to be placed on equal footing, how should we reconcile this difference in the quantum world? One possibility is that we view **particle as fundamental**, with the electromagnetic field arising only in some classical limit from a collection of quantum photons. Alternatively, we may consider **field as fundamental**, with the photon appearing only when we correctly treat the field in a manner consistent with quantum theory. In this case, we may want to introduce an analogue "electron field", whose ripples give rise to particles with mass and charge. But why then didn't Faraday, Maxwell and other classical physicists find it useful to introduce the concept of matter fields, analogous to the electromagnetic field?

In this course, we will take the second viewpoint, namely that **the field is primary and particles are derived concepts, appearing only after quantization**. We will show how photons arise from the quantization of the electromagnetic field and how massive, charged particles such as electrons arise from the quantization of matter fields. We will learn that in order to describe the fundamental laws of Nature, we must in fact introduce a field to *every* fundamental particle in nature: not only introduce electron fields, but also quark fields, neutrino fields, gluon fields, W and Z-boson fields, Higgs fields and a whole slew of others.

The advantage of the **field is fundamental** view point can be understood as due to the following reasons: (1) locality; (2) the ability to create and annihilate particles; and (3) the fact that all particles of the same type are identical.

3. Locality

One key concept which I am afraid is not properly emphasized is the concept that the laws of nature are **local**. The old laws of Coulomb and Newton involve action at a distance. This means that the force felt by an electron (or planet) **changes immediately** if a distant proton (or star) moves. **This is wrong !!**. It is both philosophically unsatisfactory, and, more importantly, experimentally wrong - it is inconsistent with the basic concept that no signal can travel faster than the speed of light, which is finite. Thus, we are seeking a theory in which all interactions are mediated in a local fashion by the field.

4. Particle number is not conserved

Another reason to treat the field, rather than the particle as the fundamental quantity is the experimental fact that when particles interact, new particles can be created (or annihilated) - **the particle number is not conserved**. This fact is demonstrated at a daily basis in CERN and other accelerators. It is a direct consequence of the combination of quantum mechanics and special relativity.

Particles are not indestructible objects, made at the beginning of the universe and here for good. They can be created and destroyed. They are, in fact, mostly ephemeral and fleeting. This experimentally verified fact was first predicted by Dirac who understood how relativity implies the necessity of anti-particles.

We will review Dirac's argument for anti-particles later in this course, together with the better understanding that we get from viewing particles in the framework of quantum field theory. For now, we will quickly sketch the circumstances in which we expect the number of particles to change.

Consider a particle of mass m trapped in a box of size L. Heisenberg tells us that the uncertainty in the momentum is $\Delta p \geq \hbar/L$. In a relativistic setting, momentum and energy are on an equivalent footing, so we should also have an uncertainty in the energy of order $\Delta E \geq \hbar c/L$. When the uncertainty in the energy exceeds $\Delta E = 2mc^2$, we cross the barrier to pop particle anti-particle pairs out of the vacuum. We learn that particle-anti-particle pairs are expected to be important when a particle of mass m is localized within a distance of order

$$\lambda = \frac{\hbar}{mc}. \tag{2}$$

The distance λ is called *Compton wavelength.* At distances shorter than this, there is a high probability that we will detect particle- anti-particle pairs swarming around the original particle that we put in. Note that Compton wavelength is always smaller than the *de Broglie wavelength*, $\lambda_{dB} = h/|\vec{p}|$: the de Broglie wavelength indicates the distance at which the wave nature of the particle becomes apparent, while Compton wavelength is the distance at which the concept of a single pointlike particle breaks down completely.

The presence of a multitude of particles and antiparticles at short distances implies that any attempt to write down a relativistic version of the one-particle Schrödinger equation is doomed to failure. There is no mechanism in standard non-relativistic quantum mechanics to deal with changes in the particle number. We thus need a new formalism to treat states with an unspecified number of particles, as is expected in the relativistic regime. This formalism is quantum field theory (QFT).

5. All particles of the same type are the same

A third reason to treat fields as the fundamental physical quantity is that all particles of the same type are the same. This is much more serious than it initially sounds. For example, two electrons are identical in every way, regardless of where they came from and what they've been through. The same is true of every other fundamental particle.

Suppose we capture a proton from a cosmic ray which we identify as coming from a supernova lying 8 billion lightyears away. We compare this proton with one freshly minted in a particle accelerator here on Earth. And the two are exactly the same! How is this possible? Why aren't there errors in proton production? *How can two objects, manufactured so far apart in space and time, be identical in all respects?* One explanation that might be offered is that there's a sea of proton "stuff" filling the universe and when we make a proton we somehow dip our hand into this stuff and from it mould a proton. Then its not surprising that protons produced in different parts of the universe are identical: they are made of the same stuff. It turns out that this is roughly what happens. The "stuff" is the proton field or, if you look closely enough, the quark field.

In fact, there is more to this tale. Being the "same" in the quantum world is not like being the "same" in the classical world: **quantum particles that are the same are truely indistinguishable**. Swapping two particles around leaves the state completely unchanged - apart from a possible minus sign. This minus sign determines the statistics of the particle. In quantum mechanics you have to put these statistics in by hand and, to agree with experiment, should choose **Bose statistics** (no minus sign) for integer spin particles, and **Fermi statistics** (yes minus sign) for half-integer spin particles. In quantum field theory, this relationship between spin and statistics is not something that you have to put in by hand. Rather, it is a consequence of the framework.

6. What is quantum field theory?

After (hopefully) convincing you that QFT is necessary, it is time to define it. The clue is in the name: **it is the quantization of a classical field**, the most familiar example of which is the electromagnetic field. **In standard quantum mechanics, we are taught to take the classical degrees of freedom and promote them to operators acting on a Hilbert space**. In QFT, we are doing essentially the same - for fields. Thus the basic degrees of freedom in quantum field theory are **operator valued functions of space and time**. Classically, every point in space has a field value associated with it; in QFT, every point in space has an operator associated with it. These operators are going to operate on

Hilbert space.

This means that we are dealing with an *infinite number of degrees of freedom* - at least one for every point in space, and there are infinite number of such points. This infinity will come back to bite on several occasions.

It will turn out that the possible interactions in quantum field theory are governed by a few basic principles: locality, symmetry and renormalization group flow (the decoupling of short distance phenomena from physics at larger scales). These ideas make QFT a very robust framework: given a set of fields there is very often an almost unique way to couple them together.

As it turns out, QFT is an extremely useful tool not only for relativistic systems (where it is necessary), but also for non-relativistic systems with many particles, such as in condensed matter physics (where collective excitations, known as *phonons* exist). It is obviously the fundamental basis for particle physics, high energy physics, and plays a major role in quantum gravity and cosmology - as well as in pure mathematics.

7. Units and scales

Nature presents us with three fundamental dimensionful constants; the speed of light c, Planck's constant (divided by 2π) $\hbar$, and Newton's gravitation constant G. They have dimensions:

$$\begin{aligned} [c] &= LT^{-1} \\ [\hbar] &= L^2MT^{-1} \\ [G] &= L^3M^{-1}T^{-2} \end{aligned} \tag{3}$$

(L - length, T - time and M - mass). Throughout this course, we will work with “natural” units, defined by

$$c = \hbar = 1 \tag{4}$$

which allows us to express all dimensionful quantities in terms of a single scale which we choose to be **mass** or, equivalently, **energy** (since $E = mc^2$ has become $E = m$). The usual choice of energy unit is eV , **the electron volt** or, more often GeV $= 10^9$ eV or TeV $= 10^{12}$ eV . To convert the unit of energy back to a unit of length or time, we need to insert the relevant powers of c and $\hbar$. For example, the length scale λ associated to a mass m is the Compton wavelength

$$\lambda = \frac{\hbar}{mc}. \tag{5}$$

With this conversion factor, the electron mass $m_e = 5 \times 10^5$ eV translates to a length scale $\lambda_e = 3.8 \times 10^{-11}$ cm.

Throughout this course we will refer to the **dimension** of a quantity, meaning the *mass dimension.* If X has dimensions of $(\text{mass})^d$ we will write $[X] = d$. In particular, the surviving natural quantity G has dimensions $[G] = -2$ and defines a mass scale,

$$G = \frac{\hbar c}{M_p^2} = \frac{1}{M_p^2} \tag{6}$$

where $M_p \approx 1.22 \times 10^{19}$ GeV is the **Planck scale.** It corresponds to a length $l_p \approx 1.6 \times 10^{-33}\ cm$. The Planck scale is thought to be the smallest length scale that makes sense: beyond this quantum gravity effects become important and it is no longer clear that the concept of spacetime makes sense. The largest length scale we can talk of is the size of the cosmological horizon, roughly $10^{60} l_p$.

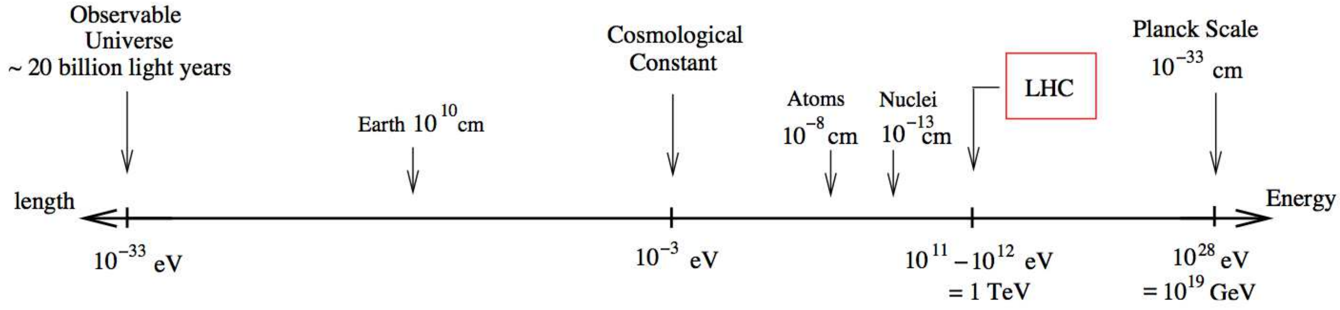

Fig. 1.— Energy and distance scales in the known universe

Some useful scales in the universe are shown in Figure 1. This is a logarithmic plot, with energy increasing to the right and, correspondingly, length increasing to the left. The smallest and largest scales known are shown on the figure, together with other relevant energy scales. The standard model of particle physics is expected to hold up to about the TeV energies. This is precisely the regime that is currently being probed by the Large Hadron Collider (LHC) at CERN. There is a general belief that the framework of quantum field theory will continue to hold to energy scales only slightly below the Planck scale - for example, there are experimental hints that the coupling constants of electromagnetism, and the weak and strong forces unify at around 10^{18} GeV.

For comparison, the rough masses of some elementary (and not so elementary) particles are shown in the table:

Particle	Mass
photon	$< 10^{-18}$ eV
neutrino	$\sim 10^{-2}$ eV
electron	0.5 MeV
Muon	100 MeV
Pions	140 MeV
Proton, Neutron	1 GeV
W,Z Bosons	80-90 GeV
Higgs Boson	125 GeV

REFERENCES

[1] D. Tong, *Lectures on Quantum Field Theory*
(http://www.damtp.cam.ac.uk/user/dt281/qft.html)