Star Formation and stellar properties

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This part of the course is based on Refs. [1] - [2].

1. Introduction

By and large, galaxies are observed through, and defined by, their stellar content. Here we describe the theory and observations of the formation of stars, as well as some of their basic properties.

Stars are believed to form from the collapse of **giant molecular clouds (GMC)**. As we saw, the baryonic gas in galaxy-sized halos cool within a time that is shorter than that of the halo. As a result, the gas loses pressure, flow towards the center of the halo potential well, while its density increases. Once the gas density exceeds the density of the dark matter, the gas continuous to collapse under its own gravitational potential.In the presence of efficient cooling, the collapse continues until matter becomes dense enough to enable the formation of stars.

1.1. Giant molecular clouds: properties

Detailed observational information about the structure of the inter-stellar medium (ISM) exists only from measurements within our own galaxy. The observations show that the gas (which contain mainly molecular hydrogen, H_2) is highly clumpy, and virtually all molecular gas is distributed in GMCs.

GMCs have typical mass of $10^5 - 10^6$ $M_\odot$, and extend over few tens of parsecs. This correspond to density of $n_{H_2} \simeq 100$ cm^{-3} (or 10^{-21} grcm^{-3}). As opposed to that, typical size of a star is $\approx 10^{11}$ cm $= 10^{-7}$ parsec, and typical density is ~ 1 grcm^{-3}. Thus, during the process of star formation, the density needs to increase by ~ 22 orders of magnitude !.

The molecular clouds usually rotate due to differential rotation in the disk in which they are formed, with typical frequency $\Omega \sim 10^{-15}$ s^{-1}. If a collapse conserves angular

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momentum, this would imply a rotation period of well below 1 s of the emerging star. This means that angular momentum has to be transferred during the collapse.

Furthermore, potential energy of the clouds, $E_p \propto -\frac{GM^2}{r}$ is released during the collapse. For a sun-size star, this corresponds to $\sim 3.8 \times 10^{48}$ erg, which is equivalent to $\sim 3 \times 10^7$ years of solar luminosity. This energy therefore must be radiated or transported away, despite the high opacities of the surrounding medium.

Observationally, the temperature of the GMCs, inferred from molecular line ratios is typically ~ 10 K. Sine the GMC show clumpy structure, where the density inside the clumps can be $\sim 10^2 - 10^4$ cm^{-3} while the temperature is approximately similar everywhere, this means that the different components inside GMCs are **not** in thermal pressure equilibrium.

Another observational fact is that GMCs show a strong spatial correlation with young star clusters, having ages of $t \leq 10^7$ yr, but little correlation with older star clusters. This indicates that the typical lifetime of a GMC is $\sim 10^7$ yr, which is much shorter than the typical age of a galaxy.

We can compare this time to the free-fall time of a GMC, which is

$$t_{\rm ff} = \left(\frac{3\pi}{32G\rho}\right)^{1/2} \simeq 3.6 \times 10^6 \text{ yr } \left(\frac{n_H}{100 \text{ cm}^{-3}}\right)^{-1/2}. \quad (1)$$

Since the free fall time is much shorter then the inferred GMC lifetime, we conclude that GMCs and their sub-clumps must be supported against gravitational collapse by some non-thermal pressure.

However, this raises the question of how do GMCs do collapse eventually to form stars? We can define the **star formation efficiency** of a GMC as

$$\epsilon_{SF} \equiv \frac{t_{\rm ff}}{t_{\rm SF}}. \quad (2)$$

Here, $t_{\rm SF} \equiv M_{GMC}/\dot{M}_\star$ is the rate at which gas turns into stars. Observationally, we know that $\epsilon SF \simeq 0.002$, though this rate changes between spiral and elliptical galaxies. For disk galaxies, $t_{\rm SF} \sim (1-5) \times 10^9$ yr $\gg t_{\rm ff}$, while for star-burst galaxies, $t_{\rm SF} \sim 10^7 - 10^8$ yr $\sim t_{\rm ff}$.

2. The Jeans mass

Similarly to the treatment we did when we discussed perturbation growth over cosmological time, we can find a lower limit on the mass of the cloud above which is will collapse due to its own gravity. This is called the **Jeans criteria**, and the critical mass is known **Jeans mass**.

Ignoring external pressure, the virial theorem states that

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 2E_k + E_p, \tag{3}$$

and collapse will occur for $2E_k + E_p < 0$.

When we deal with cloud of gas, the kinetic energy is due to the motion of the gas molecules, and we can therefore write

$$E_k = \frac{3}{2} N k_B T = \frac{3}{2} M c_s^2 \tag{4}$$

Here, N is the total number of molecules in the cloud, $c_s^2 = \left(\frac{\partial p}{\partial \rho}\right)_s = \frac{k_B T}{\mu m_p}$ is the speed of sound, and $M = \mu m_p N$ is the total mass of the cloud.

The gravitational potential energy is $E_p = -\frac{3}{5}\frac{GM^2}{r}$, and writing the density as $\rho = \frac{3M}{4\pi r^3}$, the requirement for a collapse, $2E_k < -E_p$ is translated to

$$M > M_J = \left(\frac{5c_s^2}{G}\right)^{3/2} \left(\frac{3}{4\pi\rho}\right)^{1/2} \simeq 40\ M_\odot \left(\frac{c_S}{0.2\text{km/s}}\right)^3 \left(\frac{n_{H_2}}{100\ \text{cm}^{-3}}\right)^{-1/2} \tag{5}$$

Thus, clouds with masses larger than the Jeans mass, $M > M_J$, will collapse. Clearly, a decrease n the temperature (and therefore the sound speed), or increase in the density, causes a decrease in the Jeans mass, M_J. This will result in fragmentation of the cloud into smaller clumps.

Comment: the criteria set by the Jeans mass ignores external pressure, which could be important. One can repeat the calculation for an isothermal sphere in pressure equilibrium with its environment, which would result in a **Bonner-Ebert** mass, which is given by

$$M_{BE} \simeq 1.182 \frac{c_s^3}{(G^3\rho)^{1/2}}. \tag{6}$$

Overall, this is very similar to the Jeans mass.

Thus, GMCs with $M_{GMC} \gg M_J \sim M_{BE}$ are expected to collapse on a time scale comparable to $t_{\rm ff}$ - unless the clouds have some additional support against the collapse.

3. Possible sources of non-thermal pressure

3.1. Magnetic fields

There are several possibilities for additional sources of energy that could support the GMC against collapse. One is magnetic energy. If the gas is magnetized, equating the magnetic energy, $(B^2/8\pi) \times (4\pi r^3/3)$ with the potential energy, gives a characteristic mass,

$$M_\Phi \equiv \frac{5^{3/2}}{48\pi 2^{3/2}} \frac{B^3}{G^{3/2}\rho} \simeq 10^5 \ M_\odot \ \left(\frac{n_{H_2}}{100 \ \mathrm{cm}^{-3}}\right)^{-2} \left(\frac{B}{30 \ \mu \ G}\right)^3 . \tag{7}$$

Here, it is explicitly assumed that the magnetic field is uniform across the cloud.

Similar to the thermal case, if $M > M_\Phi$, the magnetic energy cannot prevent the gravitational collapse, and the cloud will become *magnetically super-critical*; while if $M < M_\Phi$, the magnetic forces prevent the cloud from collapsing. The cloud is said to be *magnetic sub-critical*.

This raises the question, of how can low mass stars ever form? The answer is that the clouds may consist of both neutral and ionized particles; the neutral particles are affected by the magnetic field only *indirectly*, via their collision with the ionized particles. If the ionization fraction is sufficiently low, the neutral particles can diffuse through the magnetic field. This is known as **ambipolar diffusion**. The diffusion time scale is

$$t_{\mathrm{ad}} \sim 3 \times 10^7 \ \mathrm{yr} \quad \left(\frac{n_{H_2}}{100 \ \mathrm{cm}^{-3}}\right)^{3/2} \left(\frac{B}{30 \ \mu \ G}\right)^{-2} \left(\frac{R}{10 \ \mathrm{pc}}\right)^2 . \tag{8}$$

Thus, if GMCs are magnetically sub-critical, then the star formation time scale will be comparable to the ambipolar diffusion time scale. In this case, the star formation efficiency will be given by

$$\epsilon_{SF,GMC} = \frac{t_{\mathrm{ff}}}{t_{\mathrm{ad}}} \sim 0.05 - 0.1, \tag{9}$$

which is not far from the observed value.

3.2. Supersonic turbulence

An alternative (which is currently more appealing) is that GMCs are supported not by magnetic pressure, but rather by supersonic turbulence. While the description of turbulence is in general complicated, we can get an order of magnitude estimate of the effect by replacing the speed of sound in the calculation of the Jeans mass by an effective sound speed,

$$c_{s,eff} = \sqrt{c_s^2 + \frac{1}{3}\langle v^2 \rangle} = \sqrt{c_s^2 + \sigma_v^2}, \tag{10}$$

where σ_v is the non-thermal (turbulence) velocity dispersion.

Using the Jeans criteria, we see that a GMC will be stabilized against gravitational collapse if $\sigma_v \gtrsim 6$ km/s, which is roughly consistent with the observed line-width of GMCs. This fact makes this model very appealing today.

Since $\sigma_v \gg c_s \simeq 0.2$ km/s (which is determined from the observed temperature of $T = 10$ K), it is clear that the turbulentic motions must be supersonic. As the turbulentic motion is super-sonic, it drives shock waves into the plasma. A shock wave compresses the density, which scales as $\rho' = \rho\mathcal{M}^2$, where $\mathcal{M}$ is the Mach number of the flow. We thus have, in this case,

$$M_J \propto \frac{(c_s^2 + \sigma_v^2)^{3/2}}{\mathcal{M}\rho^{1/2}} \tag{11}$$

The turbulentic model has another interesting prediction. Typically, turbulence is driven at some large scale, and then the energy is spread on smaller and smaller scales, until the energy is dissipated at some small dissipation scale. On the intermediate scales, the turbulentic velocity field must have some form of power spectrum, $P_v(k) \propto k^{-n}$; this follows from the lack of any specific scale in the problem. It can be shown, that for $n = 4$, one gets $\sigma_v(R) \propto R^{1/2}$, where R is some physical scale. This result is in agreement with observations.

Using this result in Equation 11, one finds that for $\sigma_v \gg c_s$, the turbulentic motion increases the effective Jeans mass (effective pressure), thereby **preventing collapse**. This means that star formation on scale of the entire GMC is prevented.

On the other hand, for $\sigma_V < c_s$, turbulence can boost local gas compression, causing the gas on small scale to collapse. Thus, on small scales, turbulence **promote collapse**.

Various processes can contribute to the turbulentic motion. These include *external* processes to the GMC, such as: (i) galaxy mergers, tidal interactions, etc.; (ii) supernova explosions; (iii) instabilities (gravitational, thermal, MHD...); etc. There are also *internal* instabilities, such as (i) proto-stellar outflow; (ii) stellar winds; and (iii) Ionizing radiation.

4. Empirical star formation laws

Star formation rate (SFR) in galaxies are typically characterized by the mass in stars formed per unit time per unit area, namely

$$\dot{\Sigma}_\star = \frac{\dot{M}_\star}{\text{area}}. \tag{12}$$

A related quantity is the gas consumption time, $\tau_{\rm SF} \equiv \Sigma_{\rm gas}/\dot{\Sigma}_\star$.

Ideally, we would like to have a theory that will describe $\dot{\Sigma}_\star$ as a function of the relevant physical conditions in the ISM, namely density, temperature, gas metallicity, radiation field, magnetic field, etc. etc.). Unfortunately, such complete theory still does not exist. What we do have are some **empirical relations** (=relations derived from observations) between $\dot{\Sigma}_\star$ and some of the physical properties of the ISM.

Clearly, while these empirical relations are very for both development of theories and inferring the star formation rate / physical conditions in distant objects, they suffer various uncertainties, originating from incomplete observational picture.

4.1. The Kennicut-Schmidt Law

Since star formation requires gas, one naturally looks for a correlation between $\dot{\Sigma}_\star$ and the surface density of gas, $\Sigma_{\rm gas}$. A power law relation of the form

$$\dot{\Sigma}_\star \propto \Sigma_{\rm gas}^N \tag{13}$$

is known as **Schmidt law** for star formation (Schmidt, 1959).

Schmidt work was extended by Kennicutt (1998) to include starburst galaxies, where the density accounts for both atomic and molecular gas. The best fit, known as **Kennicut-Schmidt law** reads

$$\dot{\Sigma}_\star = (2.5 \pm 0.7) \times 10^{-4} \left(\frac{\Sigma_{\rm gas}}{M_\odot \ {\rm pc}^{-2}} \right)^{1.4} \ M_\odot \ {\rm yr}^{-1} \ {\rm kpc}^{-2} \tag{14}$$

It is plotted in Figure 1

A word of caution. It is very tempting to interpret the Kennicutt-Schmidt law as indicating that the star formation rate is controlled by self gravity of the gas. Indeed, in this case the star formation rate should be proportional to the gas mass, divided by the time scale for gravitational collapse. As the free fall time is $t_{\rm ff} \propto \rho_{\rm gas}^{-1/2}$ (see Equation 1) when $\rho_{\rm gas}$ is the mean gas density, one gets $\dot{\rho}_\star = \epsilon_{\rm SF} \rho_{\rm gas}/t_{\rm ff} \propto \rho_{\rm gas}^{1.5}$, where $\epsilon_{\rm SF}$ is a measure of the efficiency of star formation. If all galaxies have approximately the same scale hight, this translates to $\dot{\Sigma}_\star \propto \Sigma_{\rm gas}^{1.5}$, which is in good agreement with the empirical result. However, the catch is that the efficiency is very low, $\epsilon_{\rm SF} \ll 1$, implying that there are additional processes beyond self-gravity. Either that the time scale for star formation is indeed $t_{\rm ff}$, but in this case only a small fraction of the gas participate in the star formation process, or that the time scale for star formation is much longer than $t_{\rm ff}$. In either case, additional physics is required.

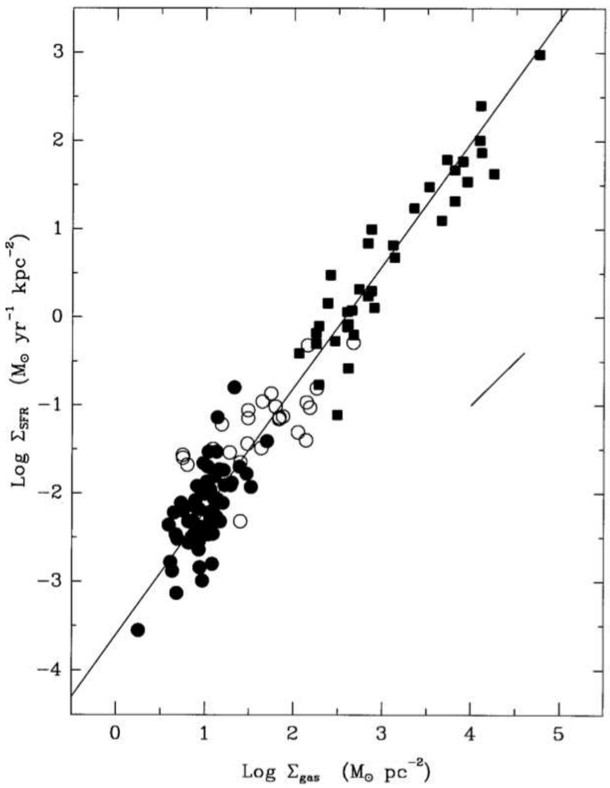

Fig. 1.— Global star formation rate per unit area as a function of the surface density of the total (atomic plus molecular) gas, $\Sigma_{\rm gas}$. Results are shown for disk galaxies (filled circles), starburst galaxies (squares), and centers of normal disk galaxies (open circles). The solid line is the Kennicutt-Schmidt law. First published by Kennicutt (1998).

5. The Initial Mass Function

The properties of stellar populations depend not only on the rate and efficiency of star formation, but also on what kind of stars are being formed. The properties and evolution of the stars depend mainly on their masses. Thus, the initial mass function (IMF), which is the initial mass spectrum within which stars are formed, is an important property that characterized star formation.

There are lower and upper limits on the masses of stars that we observe. The lower limit is $m_l \simeq 0.08\ M_\odot$. At $M < m_l$, the central temperature, which is determined by the pressure due to gravity, is too low to enable hydrogen fusion to take place. On the other extreme, stars with masses $M > m_u \simeq 100\ M_\odot$ are unstable against radiation pressure.

We can define the IMF, $\phi(m)\ dm$ as the number of stars that were *born* with masses in the range $m - dm...m + dm$, for every M_odot of newly formed stars. Thus, for every $M_\star$ of newly formed stars, the total number of stars with masses in the range $m - dm...m + dm$ is given by

$$dN(m) = \frac{M_\star}{M_\odot}\phi(m)dm \tag{15}$$

It is sometimes useful to define a logarithmic IMF by $\xi(m)d\log(m) = \phi(m)dm$, or $\xi(m) = \ln(10)m\phi(m)$.

While in general the IMF can vary between different galaxies, and even within different regions inside a galaxy, observations suggest that the variations are not large, and that the IMF is universal. Note that since we don't see the mass of a star, but only its luminosity, in estimating the mass one uses the mass-luminosity relation (which we may or may not have time to discuss). Furthermore, individual stars can practically be observed only in the milky way.

The first estimate of the IMF in the solar neighborhood was done by Salpeter (1955), and is known as **Salpeter IMF**,

$$\phi(m)dm \propto m^{-b}dm \quad b = 2.35 \tag{16}$$

for stars in the mass range $0.4m_\odot \leq M \leq 10m_\odot$.

Additional fits exist, such as the **Miller-Scalo IMF**,

$$\xi(x) = 1.53 - 0.96x + 0.47x^2, \tag{17}$$

where $x \equiv \log(m/M_\odot)$. The IMF is plotted in Figure 2.

6. Stellar properties

Stars are classical examples of systems in hydrostatic equilibrium. If we consider our sun, it has typical density $\rho \sim 1$ g cm^{-3}, radius $R \sim 7 \times 10^{10}$ cm and temperature $T \sim 7000°$ K. The typical gravitational free fall time, $t_{dyn} = (G\rho)^{-1/2}$ and the hydrodynamical time scale $t_{\rm hydro} = R/c_s$, which is the time scale for sound wave propagation, are both of the order of 1 hour.

These should be compared to the thermal emission time (the time it takes the star to radiate its energy), $t_{th} = E_{th}/L \sim 10^7$ yr, and the time scale of energy generation in the star, $t_{nuc} = \eta Mc^2/L \sim 10^{10}$ yr, where $\eta \approx 10^{-3}$ is the efficiency of conversion of rest mass to radiation. Thus, the star has plenty of time to adjust to new conditions.

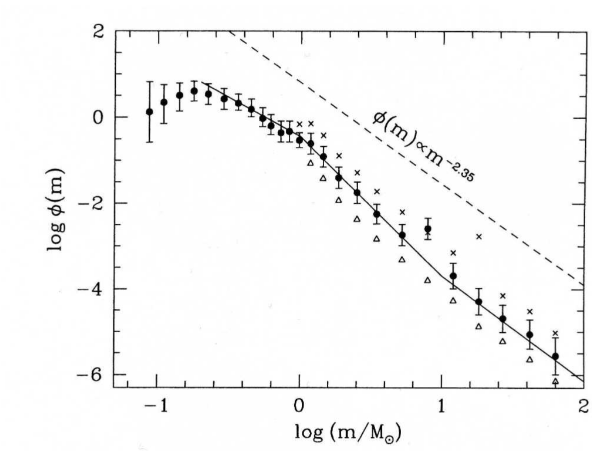

Fig. 2.— IMF or stars in the solar neighborhood. Figure taken from Scalo(1986).

6.1. Basic equations of stellar structure

Under the assumption of spherical symmetry and hydrostatic equilibrium, we have

$$\frac{dp(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}, \tag{18}$$

and

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r) \tag{19}$$

where $p(r)$, $\rho(r)$ and $M(r)$ are the pressure, density and total mass enclosed in radius r.

To close the set of equation, we need a third equation, which will be the equation of state,

$$p = p(\rho, T, \{X_i\}) \tag{20}$$

where $\{X_i\}$ is the mass fraction of elements $i = 1, 2, ...$

Since we introduced two new variables, T and $\{X_i\}$, we must add two new equations to complete the description. For the temperature, we use the fact that a temperature gradient causes an energy transport. For the moment, we consider only photon diffusion; we can write the energy flux as

$$F(r) = -\lambda \frac{dT}{dr} \tag{21}$$

where $F(r)$ is the energy flux, and have units of erg cm^{-2}s^{-}1 and λ is the transport coefficient.

It is convenient to work with the **opacity**, which is related to λ via

$$\kappa \equiv \frac{4acT^3}{3\rho\lambda} \tag{22}$$

where a is the radiation constant. The opacity κ has units of cm^2 g^{-1}. In terms of κ, the temperature gradient is written as

$$\frac{dT}{dr} = -\frac{3\kappa L\rho}{16\pi acr^2T^3} \tag{23}$$

where $L = 4\pi r^2 F$ is the luminosity (units of erg g^{-1}) at radius r. If the energy is released at rate ϵ (erg g^{-1}) at radius r, then we have

$$\frac{dL}{dr} = 4\pi r^2\rho\epsilon \tag{24}$$

We thus have a set of 4 equations (18, 19, 23 and 24) for five unknowns (ρ, M, p, T, L), which are completed by an equation of state - provided that p, κ and ϵ are known functions of $(\rho, T, \{X_i\})$.

To these, we add boundary conditions at the center $(M, L)_{r=0} = (0, 0)$ and at the stellar surface, $(p, T)_{r=r_\star} = (p_s, T_s)$. For many practical purposes, as the temperature and pressure at the surface of the star are much smaller than in the stellar interior, one may use the 'zero boundary conditions', $(p, T)_{r=r_\star} = (0, 0)$.

We can choose the mass of the star to be constant, and calculate its radius; it is thus convenient to write the structure equations using $M(r)$ as an independent variable, and set the boundary conditions as $M_\star$. When doing so, we can write the stellar equations (18, 19, 23 and 24) as

$$\frac{dr}{dM} = \frac{1}{4\pi r^2\rho(r)}, \tag{25}$$

$$\frac{dp}{dM} = -\frac{GM}{4\pi r^4}, \tag{26}$$

$$\frac{dL}{dM} = \epsilon, \tag{27}$$

$$\frac{dT}{dM} = -\frac{3\kappa L}{64\pi^2 acr^4T^3} \tag{28}$$

with the appropriate boundary conditions, $(r, L) = (0, 0)$ at $M = 0$, and $(p, T) = (P_s, T_s)$ at $M = M_\star$.

The pressure, p, opacity, κ and energy release rate ϵ result from the various nuclear processes as well as radiative processes and convection inside the star. These are functions of ρ, T and the stellar chemical composition. Once these fnctions are determined, we can integrate the stellar evolution equation.

While the details can be cumbersome, we can gain physical insight by adopting some simple scaling relations. Typically the energy production rate can be written as a power law in T,

$$\epsilon = \epsilon_0 \rho T^\eta \tag{29}$$

where $\eta = 4$ (for proton-proton reaction), and $\eta = 17$ for the CNO cycle. Using this result in Equation 27, one gets

$$L \propto \rho M T^\eta \propto \frac{M^{2+\eta}}{r^{3+\eta}} \tag{30}$$

where we used $\rho \propto M/r^3$ and $T = \frac{\mu m_p}{k_b}\frac{GM}{r}$. If we further assume a scaling law for the opacity of the form

$$\kappa \propto \rho^{\alpha-1} T^{3-\beta}, \tag{31}$$

using equation 28, together with the requirement that the temperature field should be universal to all stars, this translates into

$$r(M) \propto M^q \quad q = \frac{1+\eta+\alpha-\beta}{4+\eta+3\alpha-\beta} \tag{32}$$

This further leads to a scaling relation of the form

$$L \propto T^b \quad b = \frac{4[2-3q+(1-q)\eta]}{2-5q+(1-q)\eta} \tag{33}$$

For typical values of $\alpha \approx 2$, $\beta = 6.5$ and $\eta = 4$ for near-solar mass stars we get $b = 4.1$. For more massive stars, $\alpha = 1$, $\beta = 3$ and $\eta = 17$ resulting in $b = 8.6$. This gives a prediction of a straight line in luminosity - temperature diagram (known as H-R diagram). Furthermore, these scalings predict a luminosity - mass relation of the form $L \propto M^a$ with $a \approx 5$ for solar-mass star. As a consequence, more massive stars end their lives much faster then the less massive ones. Observations of globular clusters (in which all stars are believed to have been formed in approximately the same time) can thus provide indication of its age.

REFERENCES

[1] H. Mo, F. van den Bosch and S. White, *Galaxy Formation and Evolution* (Cambridge), chapters 9,10.

[2] Phillips, A.C., *The Physics of Stars* (wiley)

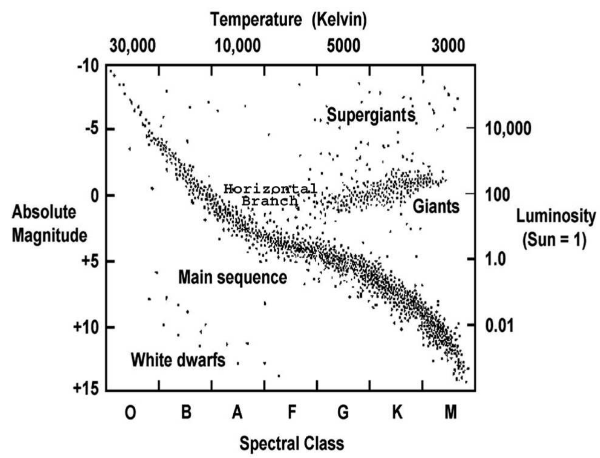

Fig. 3.— Hertzsprung-Russell (HR) diagram, showing the (log) luminosity - temperature relations.

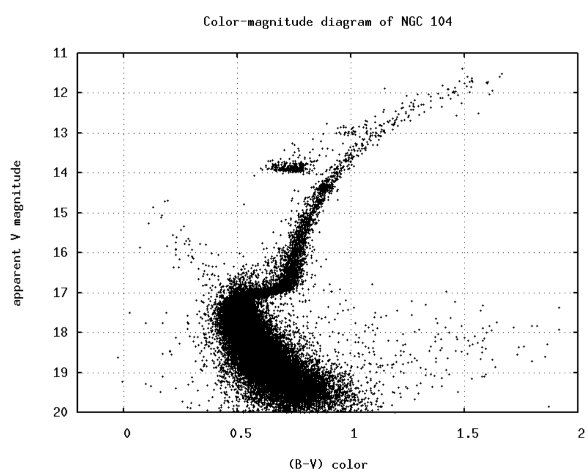

Fig. 4.— Hertzsprung-Russell (HR) diagram of the globular cluster NGC104 shows a clear turning point in the population of stars, indicating that a minimum age of the cluster.